

Probability module Day 3 at-home exercises

Please do these exercises at home between the 3rd and 4th days of Probability.

(These exercises are taken from de Groot.)

1. Suppose that a random variable X has a discrete distribution with the following probability function: $f(x) = cx$ if x is in the set $\{1, 2, 3, 4, 5\}$; otherwise $f(x)=0$. Determine the value of the constant c .

2. Each time a shopper buys a tube of toothpaste, he chooses either brand A or brand B. Suppose that for each purchase after the first, the probability is $1/3$ that he will choose the same brand he chose on his preceding purchase and the probability is $2/3$ that he will switch brands. If he is equally likely to choose either brand A or brand B on his first purchase, what is the probability that both his first and second purchases will be brand A and both his 3rd and 4th purchases will be brand B?

5. Suppose that when a machine is adjusted properly, 50% of the items it produces are of high quality and the other 50% are of medium quality. However, when it is improperly adjusted 25% of the items are of high quality and 75% are of medium quality. Suppose that the machine is improperly adjusted 10% of the time.
- a. Suppose that 5 items produced by the machine at a certain time are selected at random. Four of them are of high quality and one is of medium quality. What is the probability that the machine was properly adjusted at that time?
- b. Suppose that one additional item, which was produced by the machine at the same time as the other 5, is selected and found to be of medium quality. What is the new probability that the machine was adjusted properly?